Appendix A
RMS Values of Commonly Observed Converter Waveforms

A.1 Results for some common waveforms

See text for results

A.2 General piecewise waveform

How to compute the rms value of a waveform that can be broken into smaller piecewise segments

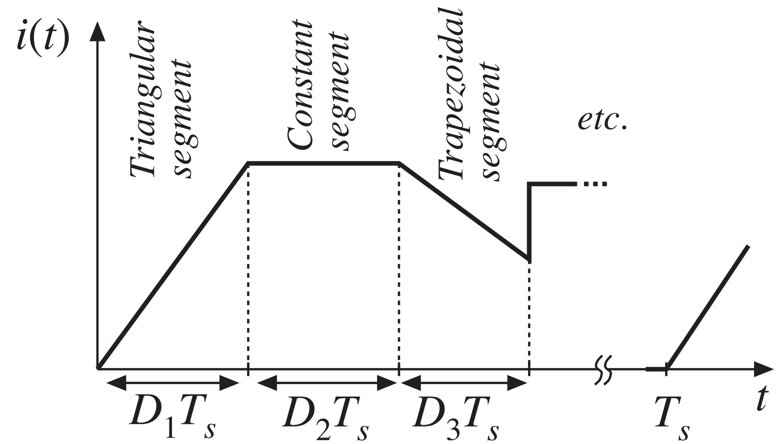
Example: transistor current waveform, including effects of short turn-on current spike

A.2 General piecewise waveform

Basic expression for rms value of waveform $v(t)$ having period T:

$$(\text{rms value}) = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

Suppose the waveform can be represented as a series of segments, with the k^{th} segment having length $D_k T_s$ and with T_s equal to the switching period:

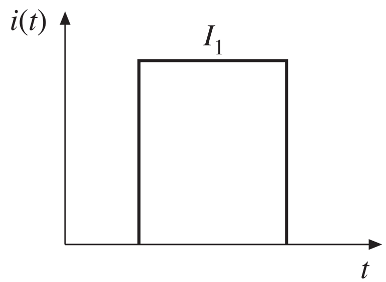


Then the rms value can be expressed as:

$$rms = \sqrt{\sum_{k=1}^{n} D_k u_k}$$

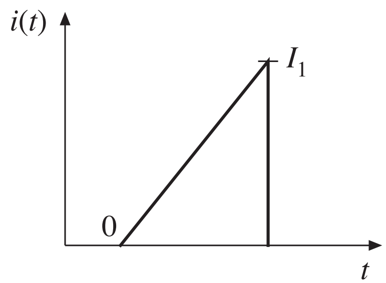
where u_k is the contribution of the k^{th} segment — see following slides

Some basic segment shapes



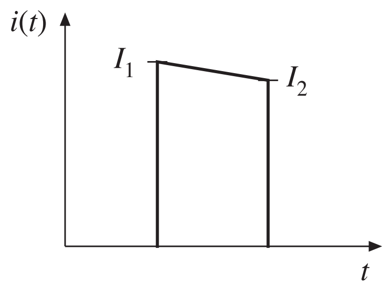
Constant segment

$$u_k = I_1^2$$



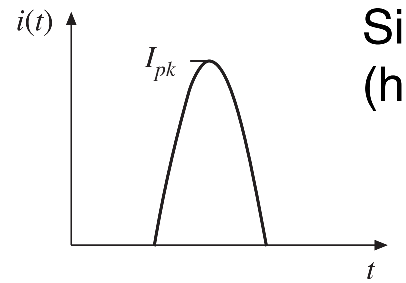
Triangular segment

$$u_k = \frac{1}{3}\, I_1^2$$



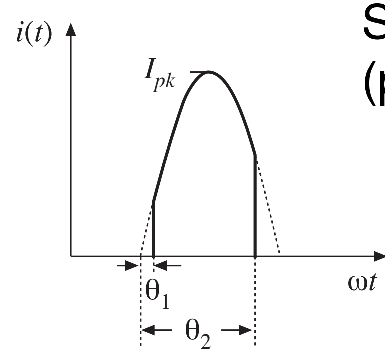
Trapezoidal segment

$$u_k = \frac{1}{3}\left(I_1^2 + I_1 I_2 + I_2^2\right)$$



Sinusoidal segment
(half or full period)

$$u_k = \frac{1}{2}\, I_{pk}^2$$



Sinusoidal segment
(partial period)

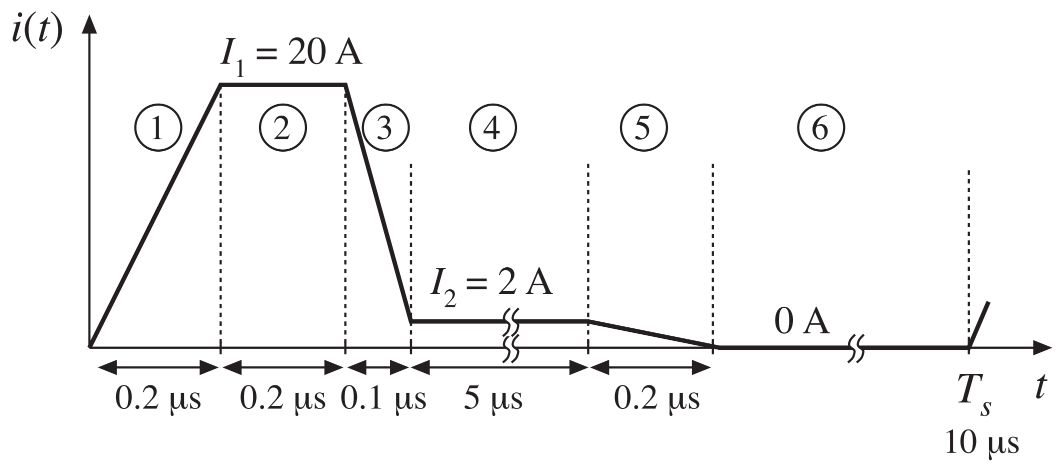
$$u_k = \frac{1}{2}\, I_{pk}^2 \left(1 - \frac{\sin\left(\theta_2 - \theta_1\right)\cos\left(\theta_2 + \theta_1\right)}{\left(\theta_2 - \theta_1\right)} \right)$$

Example

Transistor current waveform, including turn-on current spike induced by diode reverse recovery

The turn-on current spike is short but of high magnitude. Does it significantly increase the rms current?

The observed current waveform is approximated by piecewise linear segments as shown below



<u>Six segments:</u>

1-3 are from diode reverse recovery

4 is transistor on time

5 is transistor turn-off transition

6 is transistor off time

Calculation

1. Triangular segment:

$$D_1 = (0.2\ \mu s)/(10\ \mu s) = 0.02$$

$$u_1 = I_1^2/3 = (20\ A)^2/3 = 133\ A^2$$

2. Constant segment:

$$D_2 = (0.2\ \mu s)/(10\ \mu s) = 0.02$$

$$u_2 = I_1^2 = (20\ A)^2 = 400\ A^2$$

3. Trapezoidal segment:

$$D_3 = (0.1\ \mu s)/(10\ \mu s) = 0.01$$

$$u_3 = (I_1^2 + I_2^2 + I_3^2)/3 = 148\ A^2$$

4. Constant segment:

$$D_4 = (5\ \mu s)/(10\ \mu s) = 0.5$$

$$u_4 = I_2^2 = (2\ A)^2 = 4\ A^2$$

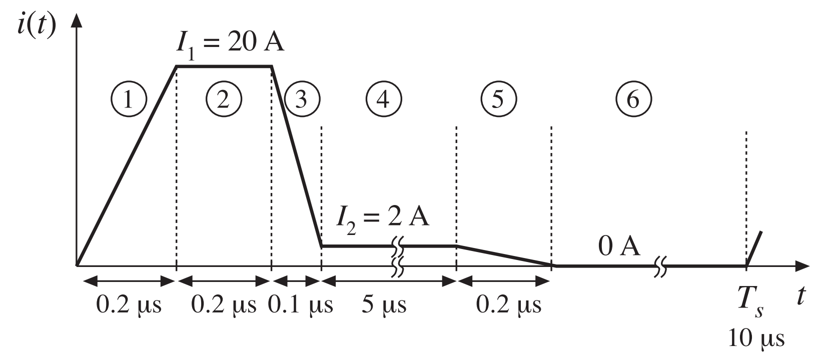
5. Triangular segment:

$$D_5 = (0.2\ \mu s)/(10\ \mu s) = 0.02$$

$$u_5 = I_2^2/3 = (2\ A)^2/3 = 1.3\ A^2$$

6. Zero segment:

$$u_6 = 0$$



Result:

$$rms = \sqrt{\sum_{k=1}^{6} D_k u_k} = 3.76\ A$$

Without the current spike, the rms value is approximately 1.4 A. So in this example, the diode reverse recovery significantly increases the transistor rms current.