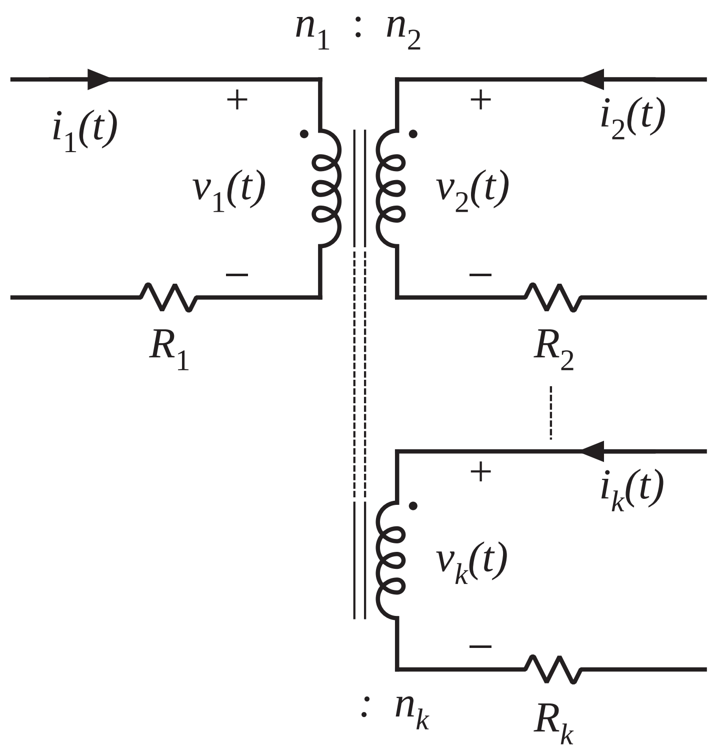
Chapter 14. Transformer Design

Some more advanced design issues, not considered in previous chapter:

- Inclusion of core loss
- Selection of operating flux density to optimize total loss
- Multiple winding design: how to allocate the available window area among several windings
- A transformer design procedure
- How switching frequency affects transformer size

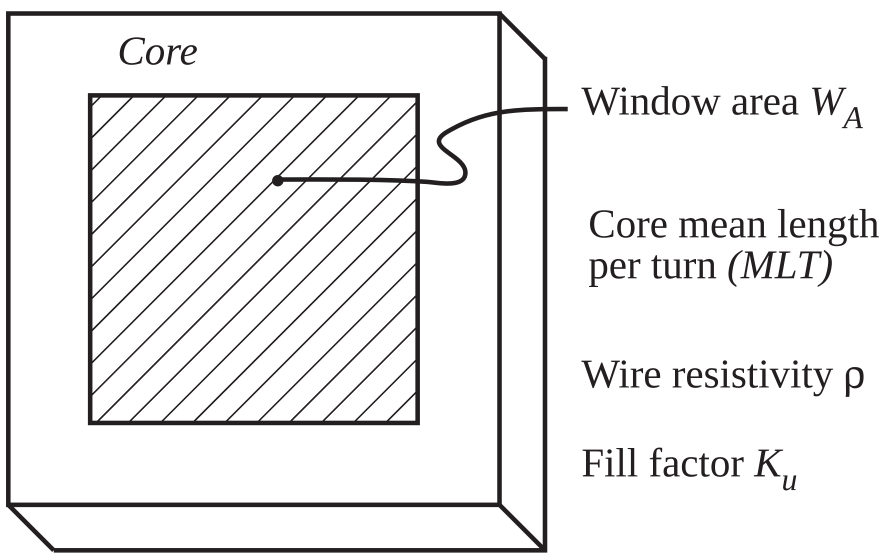


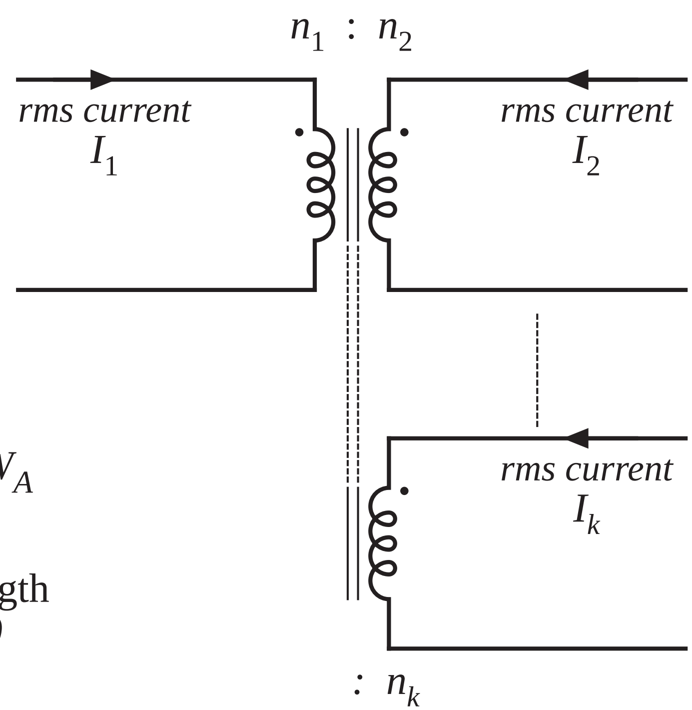
Chapter 14. Transformer Design

14.1. Winding area optimization

Given: application with k windings having known rms currents and desired turns ratios

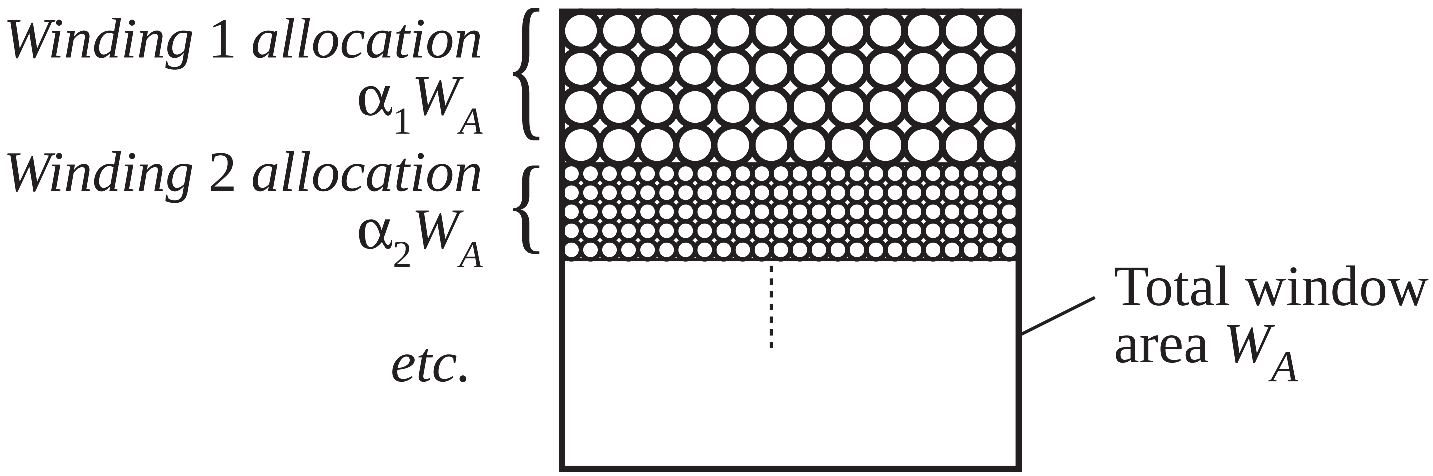
$$\frac{v_1(t)}{n_1} = \frac{v_2(t)}{n_2} = \cdots = \frac{v_k(t)}{n_k}$$





Q: how should the window area W_A be allocated among the windings?

Allocation of winding area



$$0 < \alpha_j < 1$$

$$\alpha_1 + \alpha_2 + \cdots + \alpha_k = 1$$

Copper loss in winding *j*

Copper loss (not accounting for proximity loss) is

$$P_{cu,j} = I_j^2 R_j$$

Resistance of winding *j* is

$$R_j = \rho \, \frac{l_j}{A_{W,j}}$$

with

$$l_j = n_j \, (MLT)$$ length of wire, winding *j*

$$A_{W,j} = \frac{W_A K_u \alpha_j}{n_j}$$ wire area, winding *j*

Hence

$$P_{cu,j} = \frac{n_j^2 i_j^2 \rho \, (MLT)}{W_A K_u \alpha_j}$$

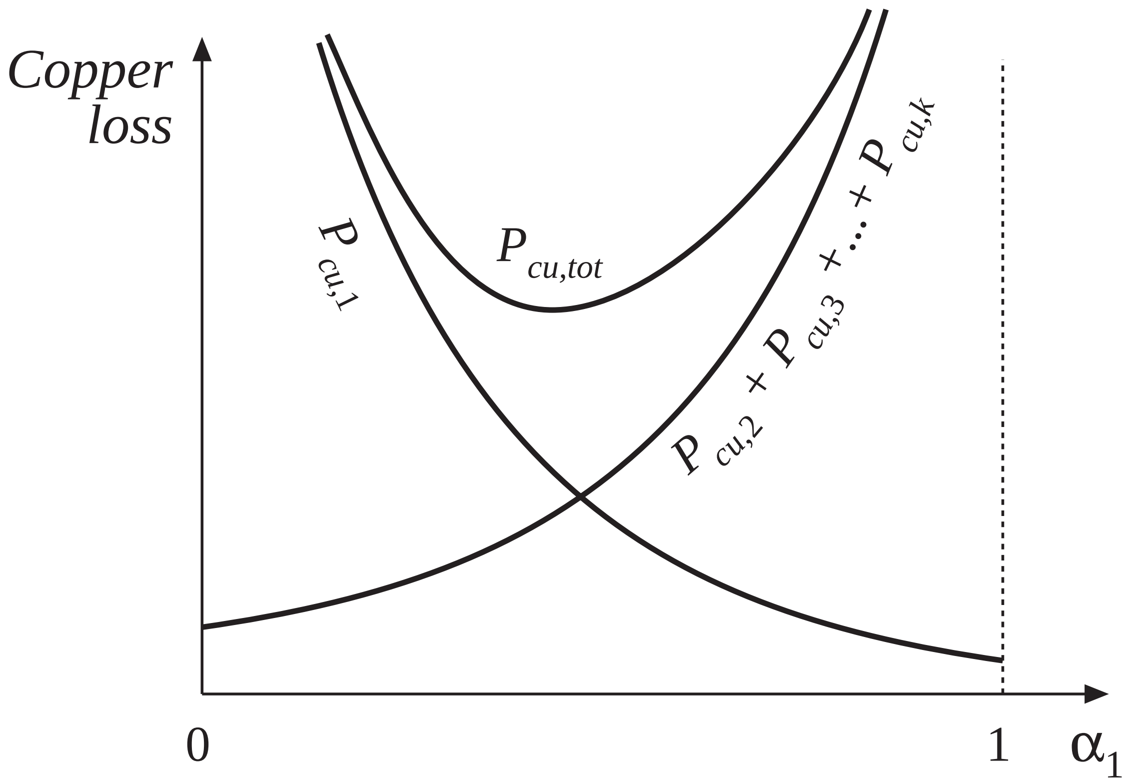
Total copper loss of transformer

Sum previous expression over all windings:

$$P_{cu,tot} = P_{cu,1} + P_{cu,2} + \cdots + P_{cu,k} = \frac{\rho\,(MLT)}{W_A K_u} \sum_{j=1}^{k} \left(\frac{n_j^2 I_j^2}{\alpha_j} \right)$$

Need to select values for $\alpha_1, \alpha_2, \ldots, \alpha_k$ such that the total copper loss is minimized

Variation of copper losses with α_1



For **α_1 = 0:** wire of winding 1 has zero area. $P_{cu,1}$ tends to infinity

For **α_1 = 1:** wires of remaining windings have zero area. Their copper losses tend to infinity

There is a choice of α_1 that minimizes the total copper loss

Method of Lagrange multipliers
to minimize total copper loss

Minimize the function

$$P_{cu,tot} = P_{cu,1} + P_{cu,2} + \cdots + P_{cu,k} = \frac{\rho\,(MLT)}{W_A K_u} \sum_{j=1}^{k} \left(\frac{n_j^2 I_j^2}{\alpha_j} \right)$$

subject to the constraint

$$\alpha_1 + \alpha_2 + \cdots + \alpha_k = 1$$

Define the function

$$f(\alpha_1, \alpha_2, \cdots, \alpha_k, \xi) = P_{cu,tot}(\alpha_1, \alpha_2, \cdots, \alpha_k) + \xi\, g(\alpha_1, \alpha_2, \cdots, \alpha_k)$$

where

$$g(\alpha_1, \alpha_2, \cdots, \alpha_k) = 1 - \sum_{j=1}^{k} \alpha_j$$

is the constraint that must equal zero

and ξ is the Lagrange multiplier

Lagrange multipliers
continued

Optimum point is solution of the system of equations

$$\frac{\partial f(\alpha_1, \alpha_2, \cdots, \alpha_k, \xi)}{\partial \alpha_1} = 0$$

$$\frac{\partial f(\alpha_1, \alpha_2, \cdots, \alpha_k, \xi)}{\partial \alpha_2} = 0$$

$$\vdots$$

$$\frac{\partial f(\alpha_1, \alpha_2, \cdots, \alpha_k, \xi)}{\partial \alpha_k} = 0$$

$$\frac{\partial f(\alpha_1, \alpha_2, \cdots, \alpha_k, \xi)}{\partial \xi} = 0$$

Result:

$$\xi = \frac{\rho\,(MLT)}{W_A K_u}\left(\sum_{j=1}^{k} n_j I_j\right)^2 = P_{cu,tot}$$

$$\alpha_m = \frac{n_m I_m}{\sum_{n=1}^{\infty} n_j I_j}$$

An alternate form:

$$\alpha_m = \frac{V_m I_m}{\sum_{n=1}^{\infty} V_j I_j}$$

Interpretation of result

$$\alpha_m = \frac{V_m I_m}{\sum_{n=1}^{\infty} V_j I_j}$$

Apparent power in winding j is

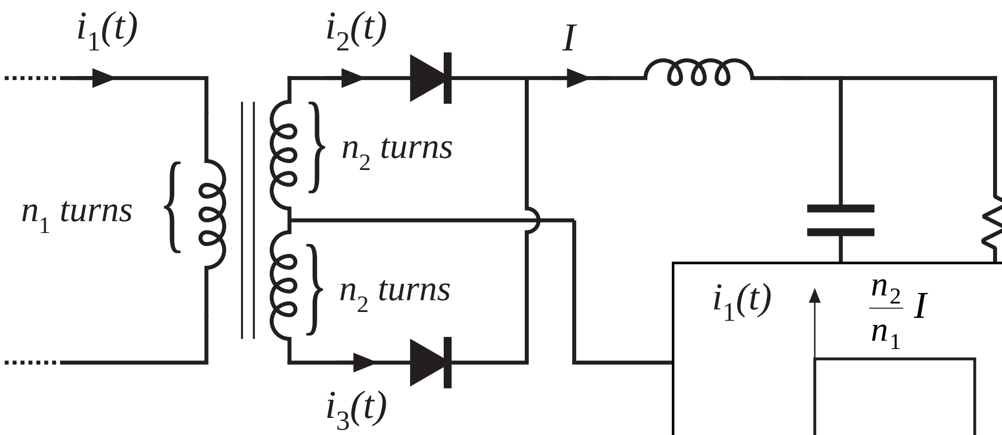
$$V_j I_j$$

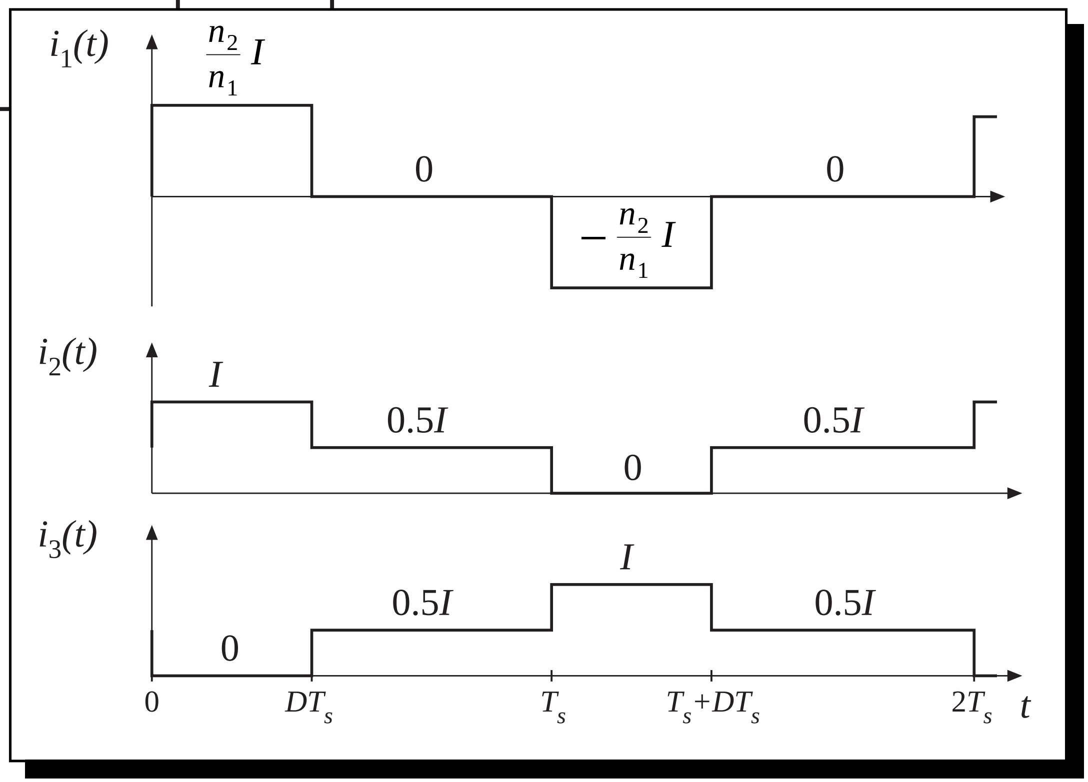
where V_j is the rms or peak applied voltage

I_j is the rms current

Window area should be allocated according to the apparent powers of the windings

Example
PWM full-bridge transformer





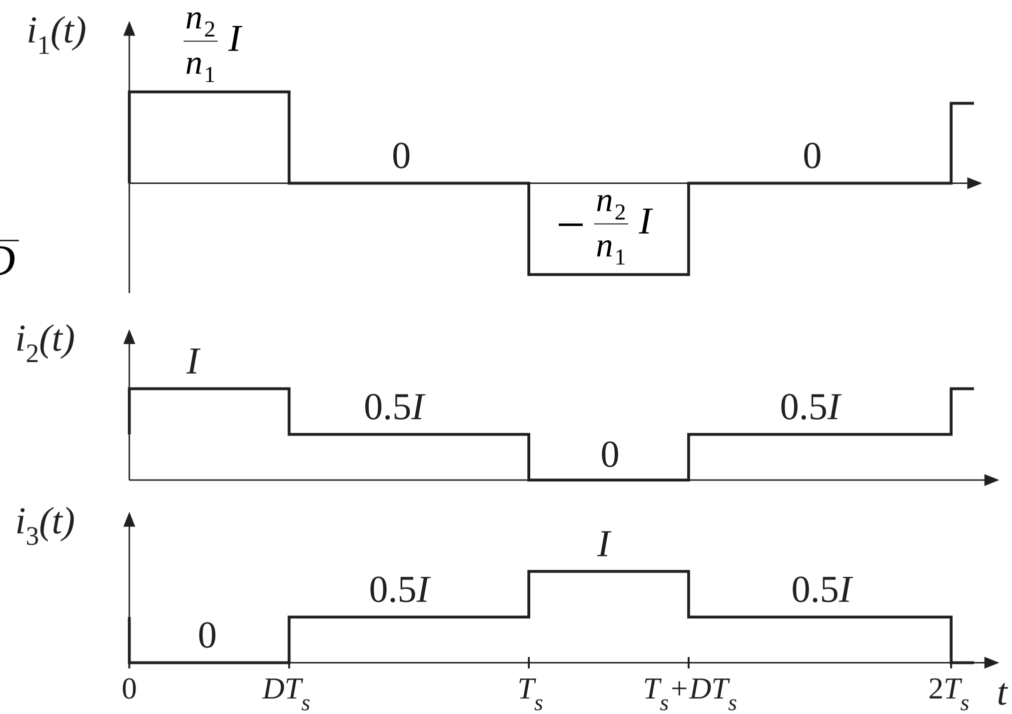
- Note that waveshapes (and hence rms values) of the primary and secondary currents are different
- Treat as a three-winding transformer

Expressions for RMS winding currents

$$I_1 = \sqrt{\frac{1}{2T_s}\int_0^{2T_s} i_1^2(t)dt} = \frac{n_2}{n_1} I \sqrt{D}$$

$$I_2 = I_3 = \sqrt{\frac{1}{2T_s}\int_0^{2T_s} i_2^2(t)dt} = \frac{1}{2} I \sqrt{1+D}$$

see Appendix 1



Allocation of window area: $\alpha_m = \frac{V_m I_m}{\sum_{n=1}^{\infty} V_j I_j}$

Plug in rms current expressions. Result:

$$\alpha_1 = \frac{1}{\left(1 + \sqrt{\frac{1 + D}{D}}\right)}$$

Fraction of window area allocated to primary winding

$$\alpha_2 = \alpha_3 = \frac{1}{2} \frac{1}{\left(1 + \sqrt{\frac{D}{1 + D}}\right)}$$

Fraction of window area allocated to each secondary winding

Numerical example

Suppose that we decide to optimize the transformer design at the worst-case operating point $D = 0.75$. Then we obtain

$$\alpha_1 = 0.396$$
$$\alpha_2 = 0.302$$
$$\alpha_3 = 0.302$$

The total copper loss is then given by

$$P_{cu,tot} = \frac{\rho(MLT)}{W_A K_u} \left(\sum_{j=1}^{3} n_j I_j \right)^2$$
$$= \frac{\rho(MLT) n_2^2 I^2}{W_A K_u} \left(1 + 2D + 2\sqrt{D(1+D)} \right)$$

14.2 Transformer design:
Basic constraints

Core loss

$$P_{fe} = K_{fe} B_{max}^{\beta} A_c l_m$$

Typical value of β for ferrite materials: 2.6 or 2.7

B_{max} is the peak value of the ac component of $B(t)$

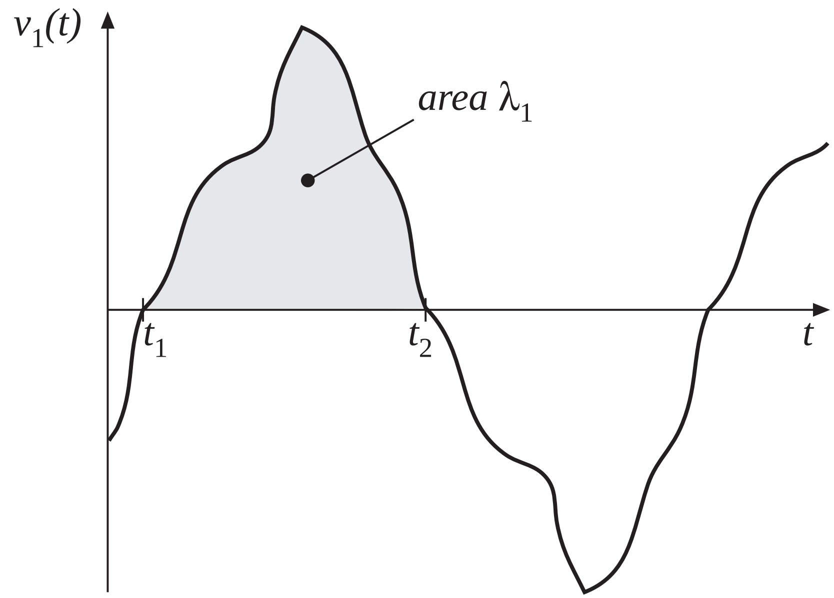
So increasing B_{max} causes core loss to increase rapidly

This is the first constraint

Flux density
Constraint #2

Flux density $B(t)$ is related to the applied winding voltage according to Faraday's Law. Denote the volt-seconds applied to the primary winding during the positive portion of $v_1(t)$ as λ_1:

$$\lambda_1 = \int_{t_1}^{t_2} v_1(t)dt$$



This causes the flux to change from its negative peak to its positive peak. From Faraday's law, the peak value of the ac component of flux density is

$$B_{max} = \frac{\lambda_1}{2n_1A_c}$$

To attain a given flux density, the primary turns should be chosen according to

$$n_1 = \frac{\lambda_1}{2B_{max}A_c}$$

Copper loss
Constraint #3

- Allocate window area between windings in optimum manner, as described in previous section
- Total copper loss is then equal to

$$P_{cu} = \frac{\rho (MLT) n_1^2 I_{tot}^2}{W_A K_u}$$

with

$$I_{tot} = \sum_{j=1}^{k} \frac{n_j}{n_1} I_j$$

Eliminate n_1, using result of previous slide:

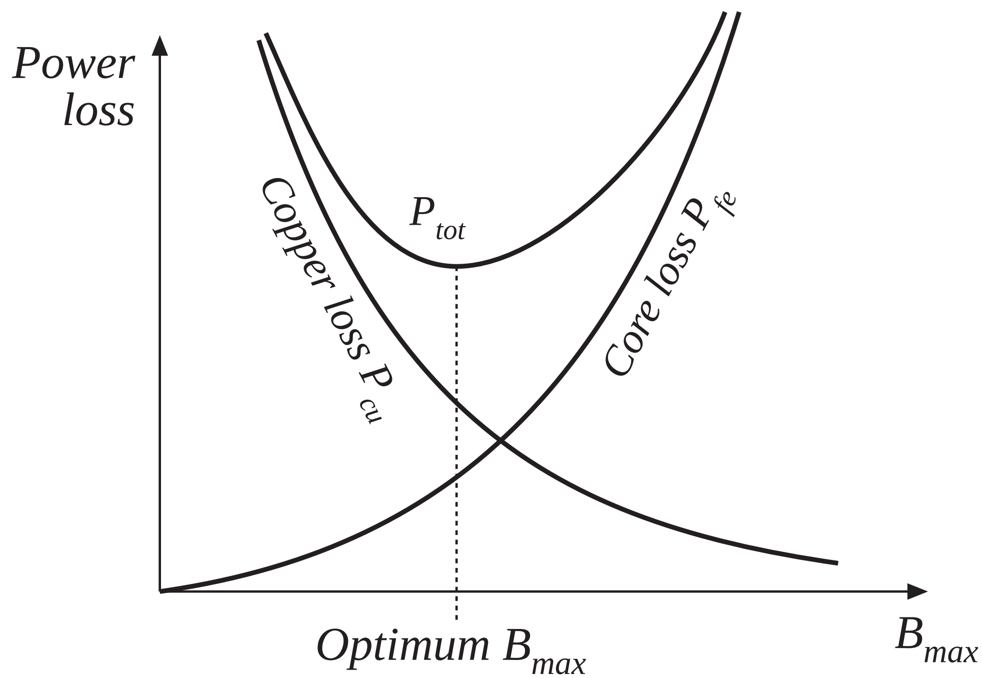
$$P_{cu} = \left(\frac{\rho \lambda_1^2 I_{tot}^2}{K_u} \right) \left(\frac{(MLT)}{W_A A_c^2} \right) \left(\frac{1}{B_{max}^2} \right)$$

Note that copper loss decreases rapidly as B_{max} is increased

Total power loss

4. $P_{tot} = P_{cu} + P_{fe}$

There is a value of B_{max} that minimizes the total power loss



$$P_{tot} = P_{fe} + P_{cu}$$

$$P_{fe} = K_{fe} B_{max}^{\beta} A_c l_m$$

$$P_{cu} = \left(\frac{\rho \lambda_1^2 I_{tot}^2}{K_u} \right) \left(\frac{(MLT)}{W_A A_c^2} \right) \left(\frac{1}{B_{max}^2} \right)$$

5. Find optimum flux density B_{max}

Given that

$$P_{tot} = P_{fe} + P_{cu}$$

Then, at the B_{max} that minimizes P_{tot}, we can write

$$\frac{dP_{tot}}{dB_{max}} = \frac{dP_{fe}}{dB_{max}} + \frac{dP_{cu}}{dB_{max}} = 0$$

Note: optimum does not necessarily occur where $P_{fe} = P_{cu}$. Rather, it occurs where

$$\frac{dP_{fe}}{dB_{max}} = -\frac{dP_{cu}}{dB_{max}}$$

Take derivatives of core and copper loss

$$P_{fe} = K_{fe} B_{max}^{\beta} A_c l_m$$

$$\frac{dP_{fe}}{dB_{max}} = \beta K_{fe} B_{max}^{(\beta - 1)} A_c l_m$$

$$P_{cu} = \left(\frac{\rho \lambda_1^2 I_{tot}^2}{K_u} \right) \left(\frac{(MLT)}{W_A A_c^2} \right) \left(\frac{1}{B_{max}^2} \right)$$

$$\frac{dP_{cu}}{dB_{max}} = -2 \left(\frac{\rho \lambda_1^2 I_{tot}^2}{4 K_u} \right) \left(\frac{(MLT)}{W_A A_c^2} \right) B_{max}^{-3}$$

Now, substitute into $\frac{dP_{fe}}{dB_{max}} = -\frac{dP_{cu}}{dB_{max}}$ and solve for B_{max}:

$$B_{max} = \left[\frac{\rho \lambda_1^2 I_{tot}^2}{2 K_u} \frac{(MLT)}{W_A A_c^3 l_m} \frac{1}{\beta K_{fe}} \right]^{\left(\frac{1}{\beta + 2}\right)}$$

Optimum B_{max} for a given core and application

Total loss

Substitute optimum B_{max} into expressions for P_{cu} and P_{fe}. The total loss is:

$$P_{tot} = \left[A_c l_m K_{fe}\right]^{\left(\frac{2}{\beta+2}\right)} \left[\frac{\rho \lambda_1^2 I_{tot}^2}{4K_u} \frac{(MLT)}{W_A A_c^2}\right]^{\left(\frac{\beta}{\beta+2}\right)} \left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)}\right]$$

Rearrange as follows:

$$\frac{W_A \left(A_c\right)^{(2(\beta-1)/\beta)}}{(MLT)\, l_m^{(2/\beta)}} \left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)}\right]^{-\left(\frac{\beta+2}{\beta}\right)} = \frac{\rho \lambda_1^2 I_{tot}^2 K_{fe}^{(2/\beta)}}{4K_u \left(P_{tot}\right)^{((\beta+2)/\beta)}}$$

Left side: terms depend on core geometry

Right side: terms depend on specifications of the application

The core geometrical constant K_{gfe}

Define
$$K_{gfe} = \frac{W_A \left(A_c\right)^{\left(2(\beta-1)/\beta\right)}}{(MLT)\, l_m^{\left(2/\beta\right)}} \left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)} \right]^{-\left(\frac{\beta+2}{\beta}\right)}$$

Design procedure: select a core that satisfies
$$K_{gfe} \geq \frac{\rho \lambda_1^2 I_{tot}^2 K_{fe}^{\left(2/\beta\right)}}{4 K_u \left(P_{tot}\right)^{\left(\left(\beta+2\right)/\beta\right)}}$$

Appendix 2 lists the values of K_{gfe} for common ferrite cores

K_{gfe} is similar to the K_g geometrical constant used in Chapter 13:

- K_g is used when B_{max} is specified
- K_{gfe} is used when B_{max} is to be chosen to minimize total loss

14.3 Step-by-step transformer design procedure

The following quantities are specified, using the units noted:

Wire effective resistivity	ρ	(Ω-cm)
Total rms winding current, ref to pri	I_{tot}	(A)
Desired turns ratios	n_2/n_1, n_3/n_1, etc.	
Applied pri volt-sec	λ_1	(V-sec)
Allowed total power dissipation	P_{tot}	(W)
Winding fill factor	K_u	
Core loss exponent	β	
Core loss coefficient	K_{fe}	(W/cm^{3}T$^\beta$)

Other quantities and their dimensions:

Core cross-sectional area	A_c	(cm^2)
Core window area	W_A	(cm^2)
Mean length per turn	MLT	(cm)
Magnetic path length	l_e	(cm)
Wire areas	A_{w1}, …	(cm^2)
Peak ac flux density	B_{max}	(T)

Procedure

1. Determine core size

$$K_{gfe} \geq \frac{\rho \lambda_1^2 I_{tot}^2 K_{fe}^{(2/\beta)}}{4 K_u \left(P_{tot}\right)^{\left((\beta+2)/\beta\right)}} 10^8$$

Select a core from Appendix 2 that satisfies this inequality.

It may be possible to reduce the core size by choosing a core material that has lower loss, i.e., lower K_{fe}.

2. Evaluate peak ac flux density

$$B_{max} = \left[10^8 \frac{\rho \lambda_1^2 I_{tot}^2}{2K_u} \frac{(MLT)}{W_A A_c^3 l_m} \frac{1}{\beta K_{fe}} \right]^{\left(\frac{1}{\beta + 2} \right)}$$

At this point, one should check whether the saturation flux densityis exceeded. If the core operates with a flux dc bias B_{dc}, then $B_{max} + B_{dc}$ should be less than the saturation flux density.

If the core will saturate, then there are two choices:

- Specify B_{max} using the K_g method of Chapter 13, or
- Choose a core material having greater core loss, then repeat steps 1 and 2

3. and 4. Evaluate turns

Primary turns:

$$n_1 = \frac{\lambda_1}{2B_{max}A_c} \; 10^4$$

Choose secondary turns according to desired turns ratios:

$$n_2 = n_1 \left(\frac{n_2}{n_1} \right)$$

$$n_3 = n_1 \left(\frac{n_3}{n_1} \right)$$

$$\vdots$$

5. and 6. Choose wire sizes

Fraction of window area assigned to each winding:

$$\alpha_1 = \frac{n_1 I_1}{n_1 I_{tot}}$$

$$\alpha_2 = \frac{n_2 I_2}{n_1 I_{tot}}$$

$$\vdots$$

$$\alpha_k = \frac{n_k I_k}{n_1 I_{tot}}$$

Choose wire sizes according to:

$$A_{w1} \le \frac{\alpha_1 K_u W_A}{n_1}$$

$$A_{w2} \le \frac{\alpha_2 K_u W_A}{n_2}$$

$$\vdots$$

Check: computed transformer model

Predicted magnetizing inductance, referred to primary:

$$L_M = \frac{\mu n_1^2 A_c}{l_m}$$

Peak magnetizing current:

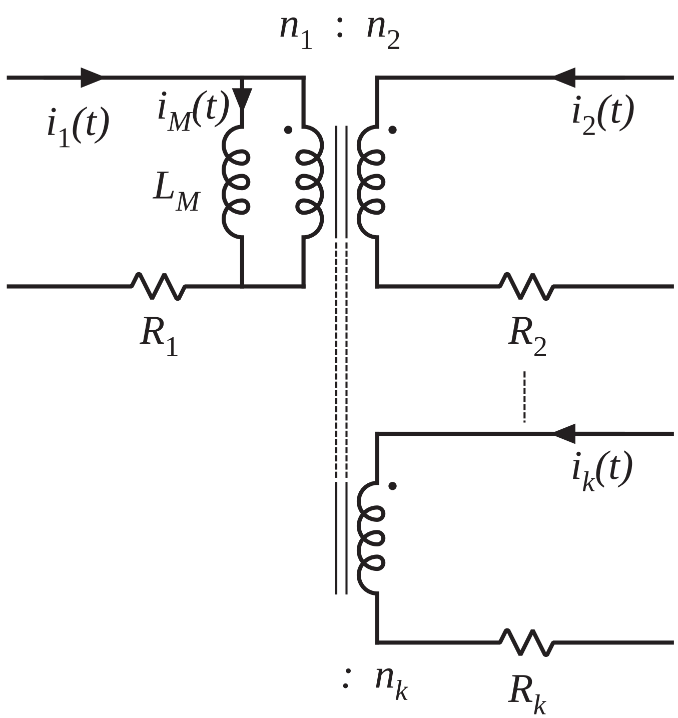
$$i_{M,\,pk} = \frac{\lambda_1}{2L_M}$$

Predicted winding resistances:

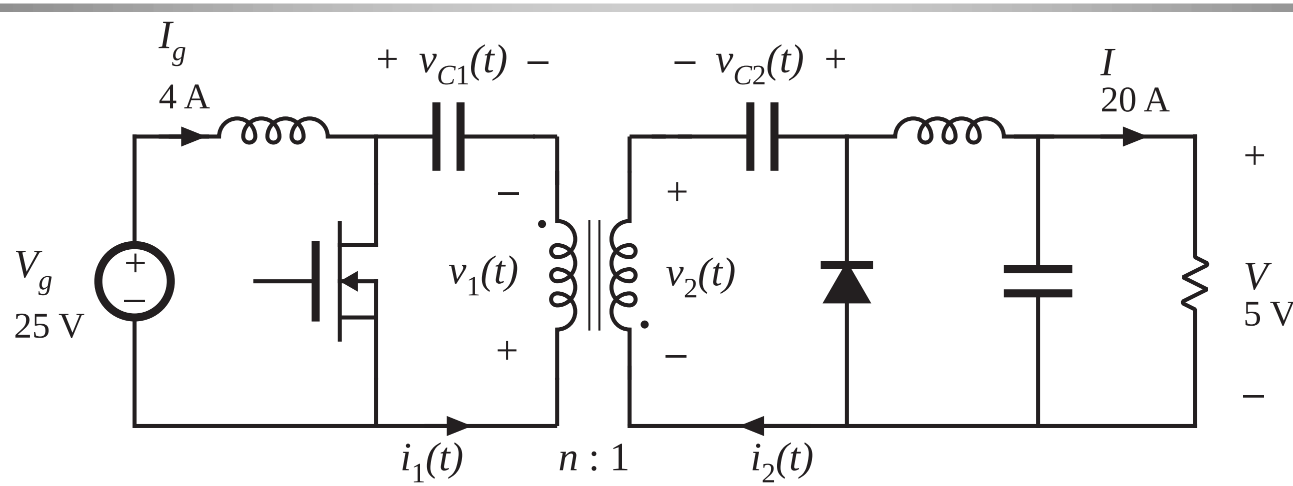
$$R_1 = \frac{\rho n_1 (MLT)}{A_{w1}}$$

$$R_2 = \frac{\rho n_2 (MLT)}{A_{w2}}$$

$$\vdots$$



14.4.1 Example 1: Single-output isolated Cuk converter



100 W

f_s = 200 kHz

D = 0.5

n = 5

K_u = 0.5

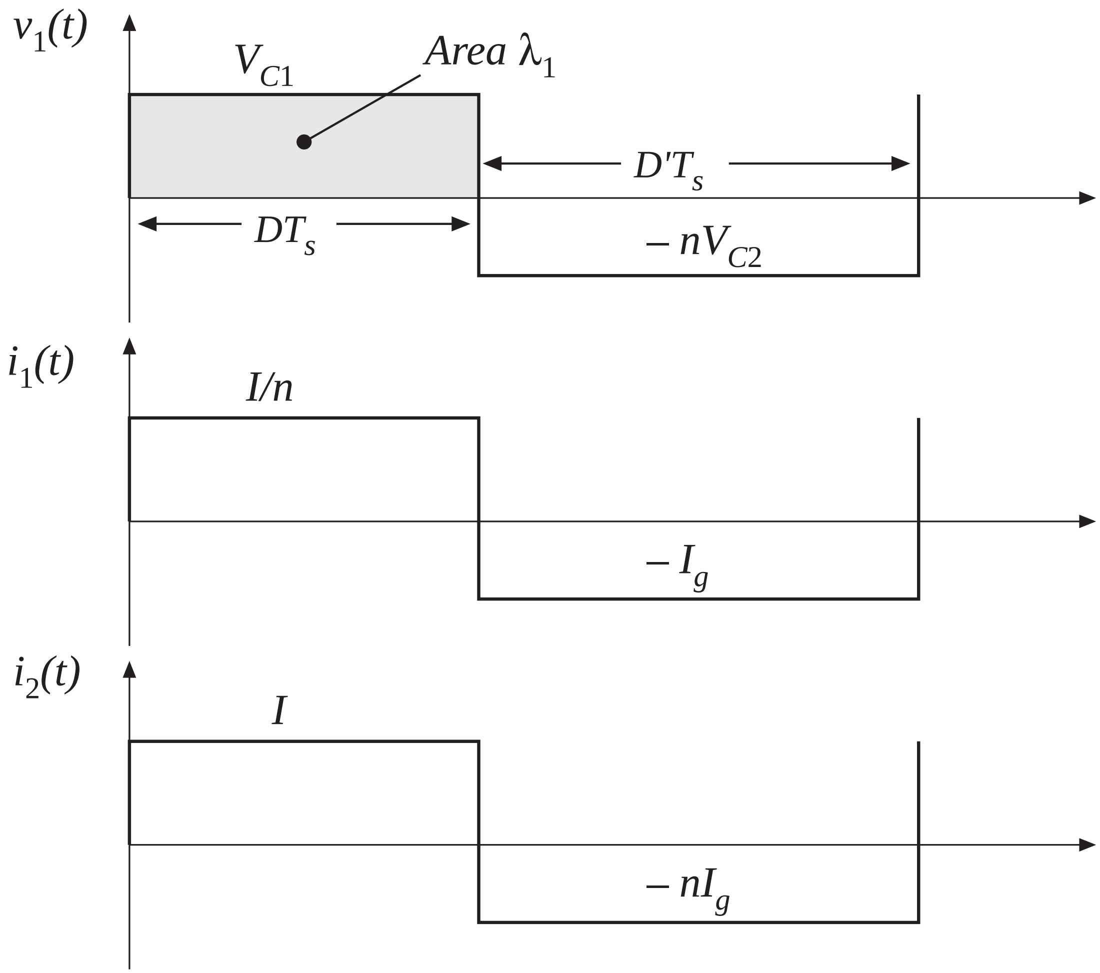
Allow P_{tot} = 0.25 W

Use a ferrite pot core, with Magnetics Inc. P material. Loss parameters at 200 kHz are

K_{fe} = 24.7

β = 2.6

Waveforms



Applied primary volt-seconds:

$$\lambda_1 = DT_sV_{c1} = (0.5)\,(5\ \mu\text{sec})\,(25\ \text{V}) = 62.5\ \text{V–}\mu\text{sec}$$

Applied primary rms current:

$$I_1 = \sqrt{D\left(\frac{I}{n}\right)^2 + D'\left(I_g\right)^2} = 4\ \text{A}$$

Applied secondary rms current:

$$I_2 = nI_1 = 20\ \text{A}$$

Total rms winding current:

$$I_{tot} = I_1 + \frac{1}{n}\, I_2 = 8\ \text{A}$$

Choose core size

$$K_{gfe} \geq \frac{(1.724\cdot 10^{-6})(62.5\cdot 10^{-6})^2(8)^2(24.7)^{(2/2.6)}}{4\,(0.5)\,(0.25)^{(4.6/2.6)}}\;10^8$$
$$= 0.00295$$

Pot core data of Appendix 2 lists 2213 pot core with

$$K_{gfe} = 0.0049$$

Next smaller pot core is not large enough.

Evaluate peak ac flux density

$$B_{max} = \left[10^8 \frac{(1.724\cdot 10^{-6})(62.5\cdot 10^{-6})^2(8)^2}{2\,(0.5)} \frac{(4.42)}{(0.297)(0.635)^3(3.15)} \frac{1}{(2.6)(24.7)} \right]^{(1/4.6)}$$

$$= 0.0858 \text{ Tesla}$$

This is much less than the saturation flux density of approximately 0.35 T. Values of Bmax in the vicinity of 0.1 T are typical for ferrite designs that operate at frequencies in the vicinity of 100 kHz.

Evaluate turns

$$n_1 = 10^4 \frac{(62.5 \cdot 10^{-6})}{2(0.0858)(0.635)}$$
$$= 5.74 \text{ turns}$$

$$n_2 = \frac{n_1}{n} = 1.15 \text{ turns}$$

In practice, we might select

$n_1 = 5$ and $n_2 = 1$

This would lead to a slightly higher flux density and slightly higher loss.

Determine wire sizes

Fraction of window area allocated to each winding:

$$\alpha_1 = \frac{(4\ \text{A})}{(8\ \text{A})} = 0.5$$

$$\alpha_2 = \frac{\left(\frac{1}{5}\right)(20\ \text{A})}{(8\ \text{A})} = 0.5$$

(Since, in this example, the ratio of winding rms currents is equal to the turns ratio, equal areas are allocated to each winding)

Wire areas:

From wire table, Appendix 2:

$$A_{w1} = \frac{(0.5)(0.5)(0.297)}{(5)} = 14.8 \cdot 10^{-3}\ \text{cm}^2$$

AWG #16

$$A_{w2} = \frac{(0.5)(0.5)(0.297)}{(1)} = 74.2 \cdot 10^{-3}\ \text{cm}^2$$

AWG #9

Wire sizes: discussion

Primary

5 turns #16 AWG

Secondary

1 turn #9 AWG

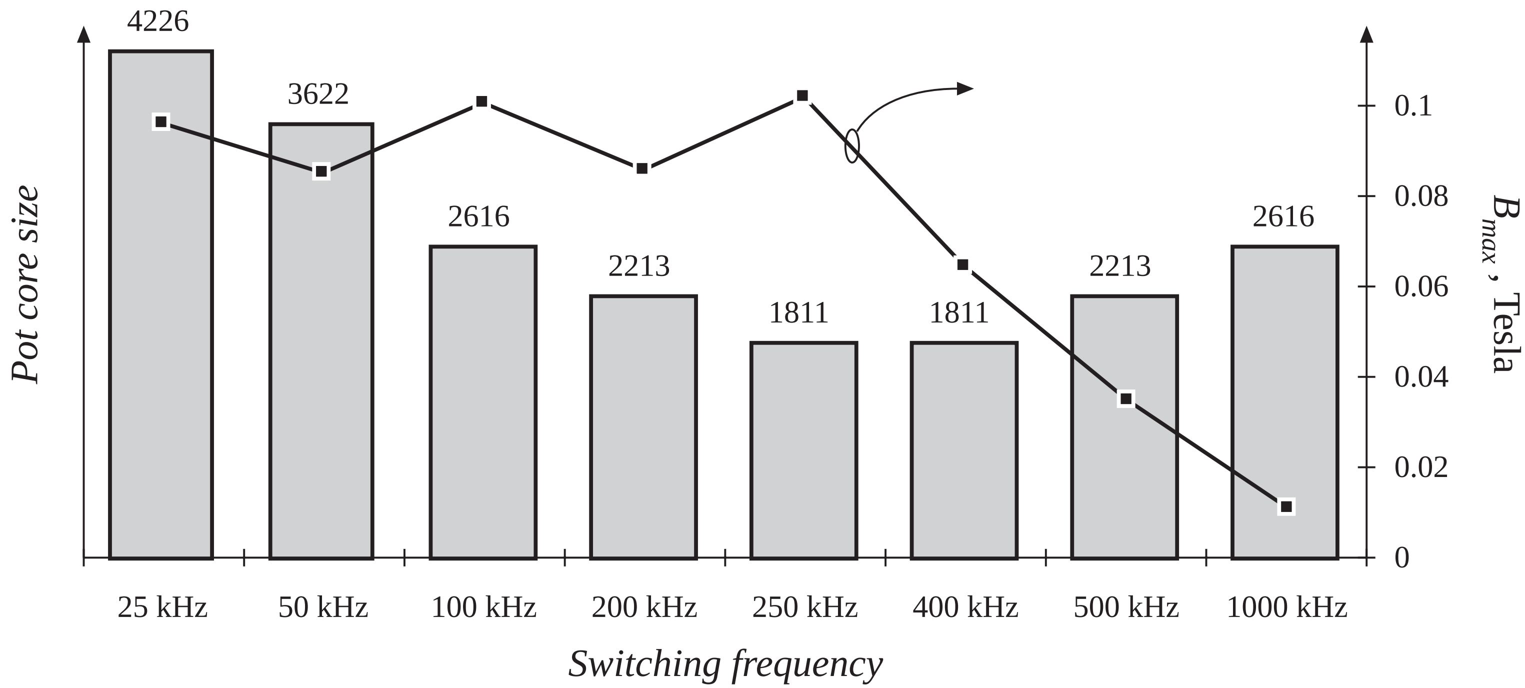
- Very large conductors!
- One turn of #9 AWG is not a practical solution

Some alternatives

- Use foil windings
- Use Litz wire or parallel strands of wire

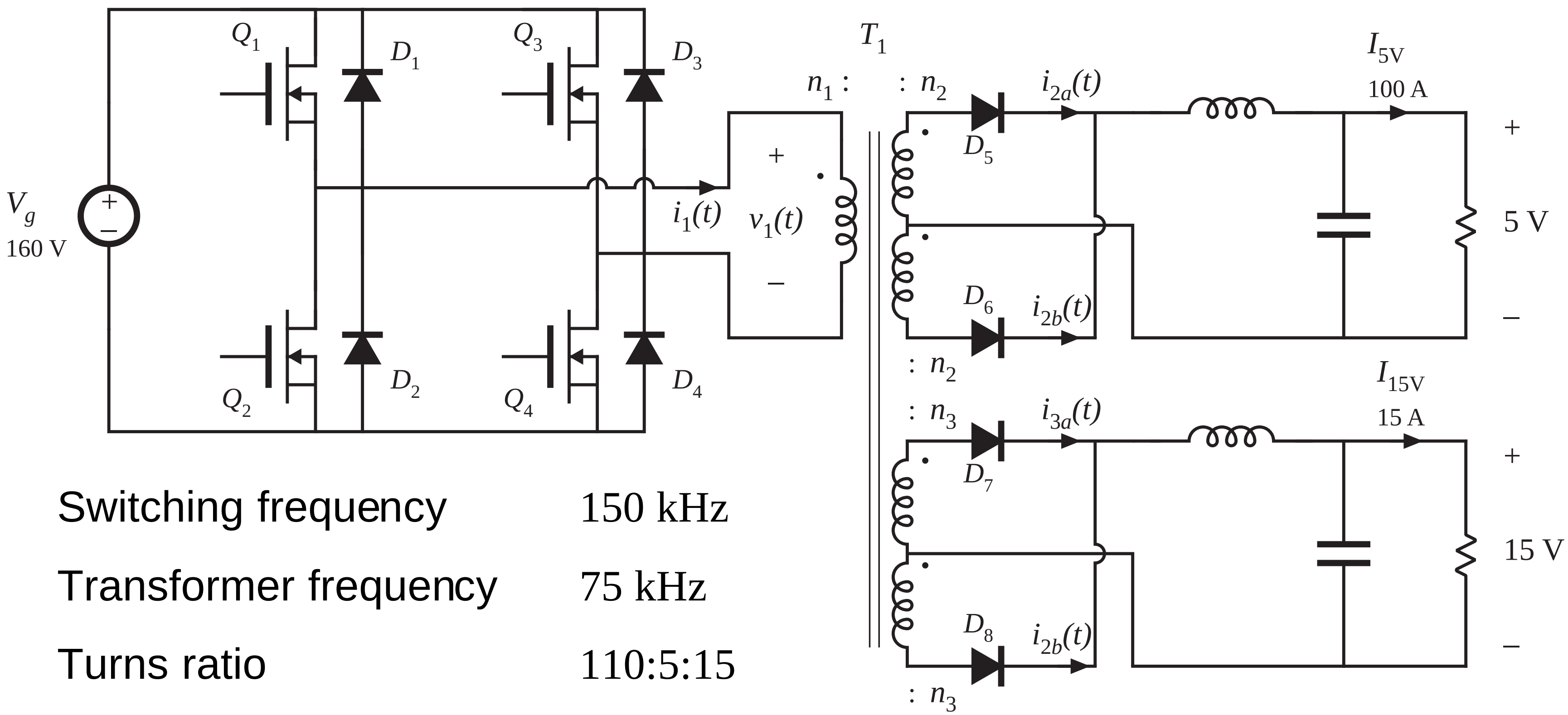
Effect of switching frequency on transformer size

for this P-material Cuk converter example



- As switching frequency is increased from 25 kHz to 250 kHz, core size is dramatically reduced
- As switching frequency is increased from 400 kHz to 1 MHz, core size increases

14.4.2 Example 2
Multiple-Output Full-Bridge Buck Converter



Switching frequency	150 kHz
Transformer frequency	75 kHz
Turns ratio	110:5:15
Optimize transformer at	D = 0.75

Other transformer design details

Use Magnetics, Inc. ferrite P material. Loss parameters at 75 kHz:

$K_{fe} = 7.6$ W/T$^{\beta}$cm^3

$\beta = 2.6$

Use E-E core shape

Assume fill factor of

$K_u = 0.25$ (reduced fill factor accounts for added insulation required in multiple-output off-line application)

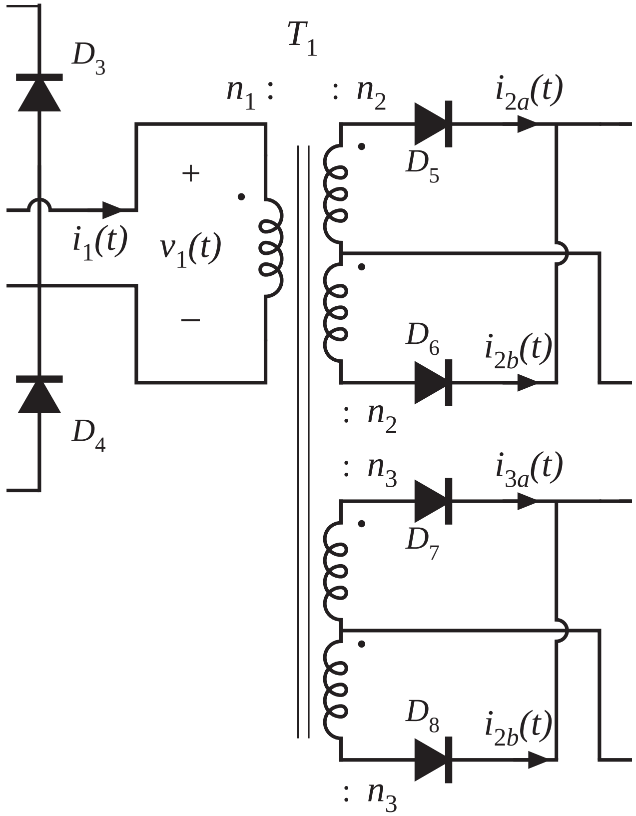
Allow transformer total power loss of

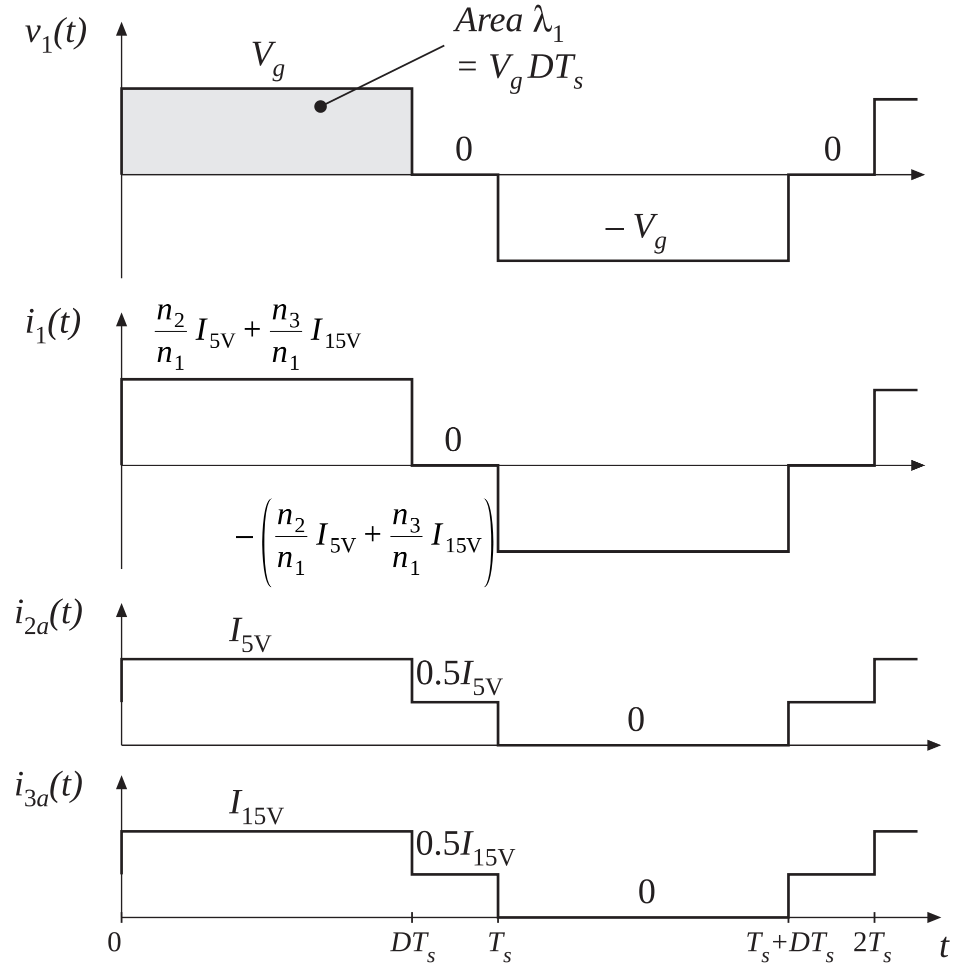
$P_{tot} = 4$ W (approximately 0.5% of total output power)

Use copper wire, with

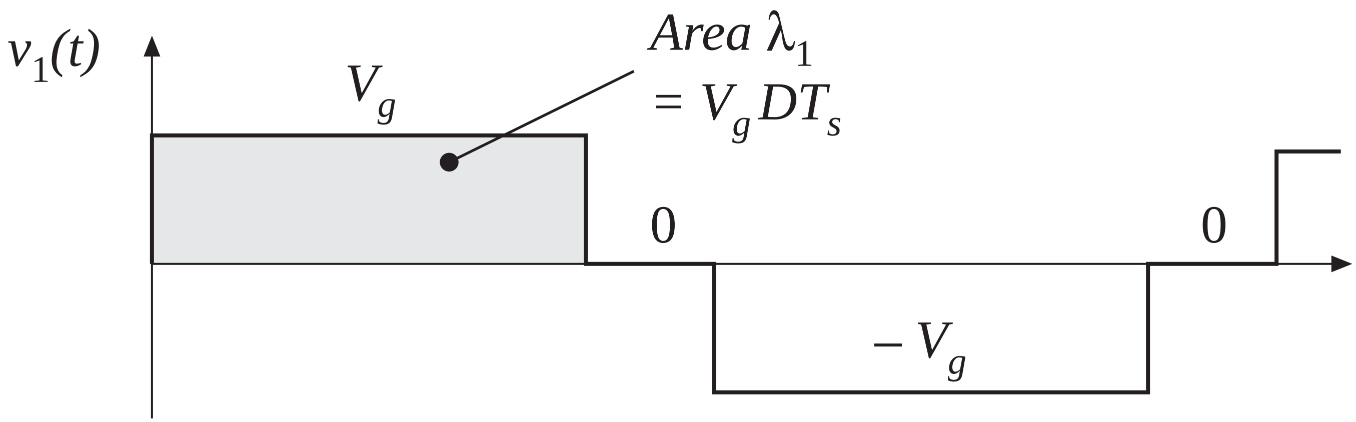
$\rho = 1.724 \cdot 10^{-6}$ Ω-cm

Applied transformer waveforms



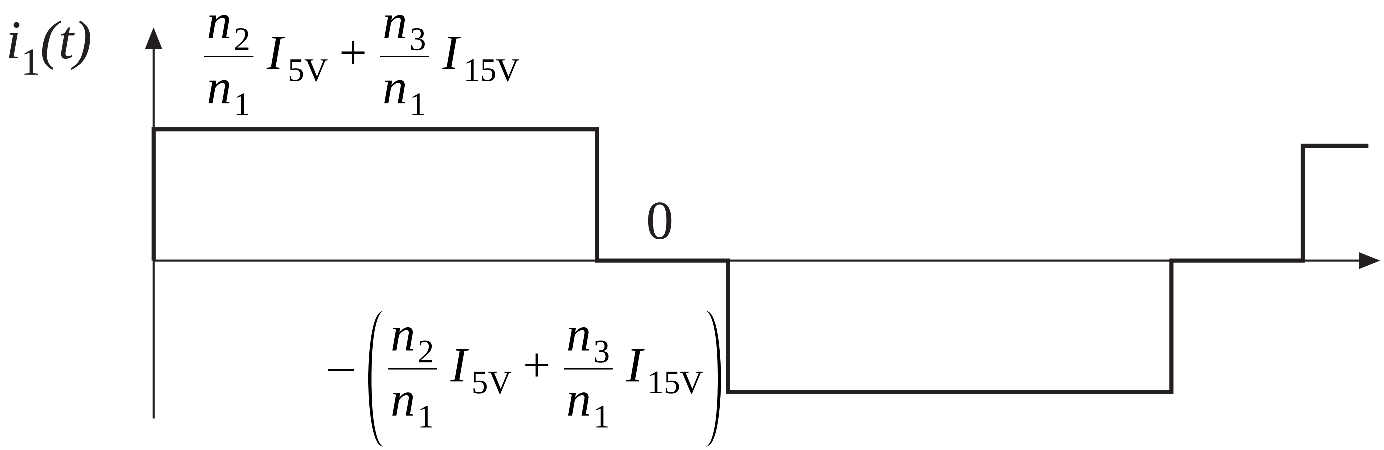


Applied primary volt-seconds



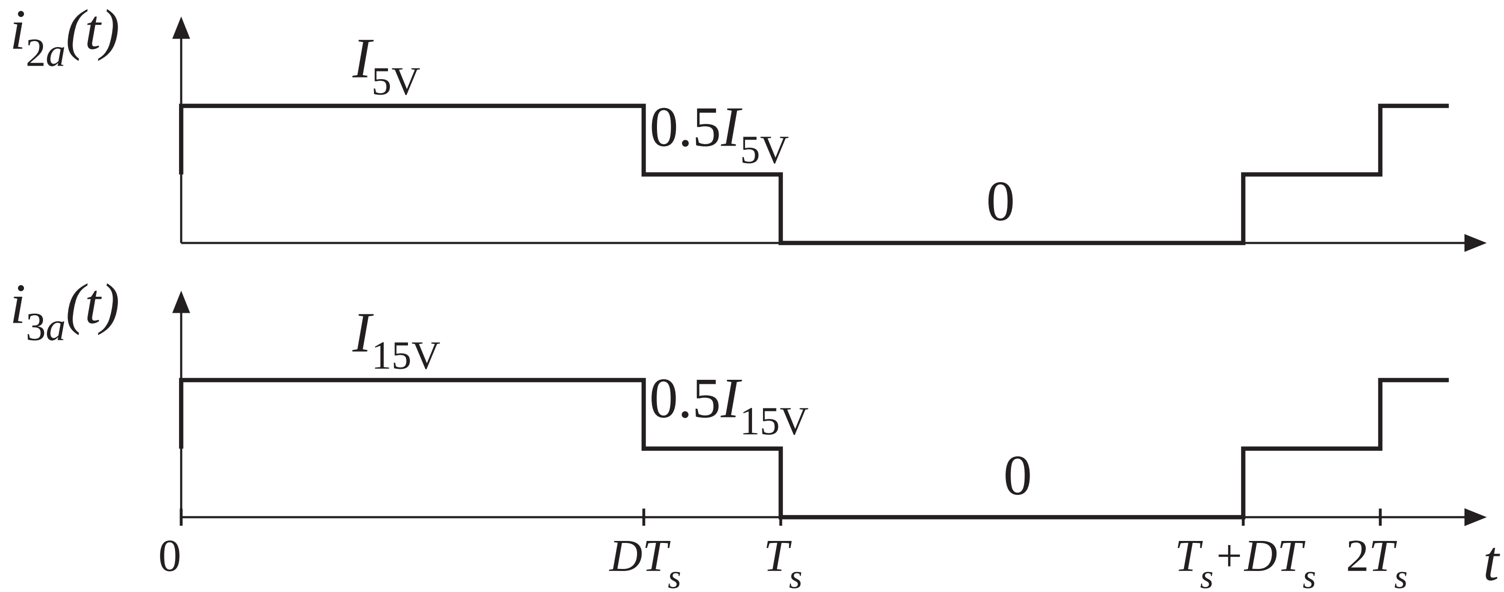
$$\lambda_1 = DT_sV_g = (0.75)\,(6.67\ \mu\text{sec})\,(160\ \text{V}) = 800\ \text{V–}\mu\text{sec}$$

Applied primary rms current



$$I_1 = \left(\frac{n_2}{n_1} I_{5V} + \frac{n_3}{n_1} I_{15V}\right)\sqrt{D} = 5.7 \text{ A}$$

Applied rms current, secondary windings



$$I_2 = \frac{1}{2}\, I_{5V} \sqrt{1+D} = 66.1 \text{ A}$$

$$I_3 = \frac{1}{2}\, I_{15V} \sqrt{1+D} = 9.9 \text{ A}$$

I_{tot}

RMS currents, summed over all windings and referred to primary

$$I_{tot} = \sum_{\substack{all\ 5 \\ windings}} \frac{n_j}{n_1} I_j = I_1 + 2\,\frac{n_2}{n_1} I_2 + 2\,\frac{n_3}{n_1} I_3$$

$$= \left(5.7\ \mathrm{A}\right) + \frac{5}{110}\left(66.1\ \mathrm{A}\right) + \frac{15}{110}\left(9.9\ \mathrm{A}\right)$$

$$= 14.4\ \mathrm{A}$$

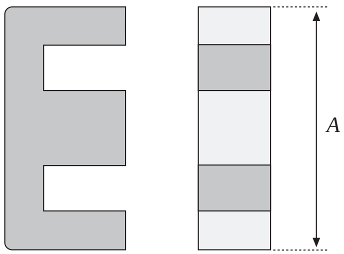
Select core size

$$K_{gfe} \ge \frac{(1.724 \cdot 10^{-6})(800 \cdot 10^{-6})^2 (14.4)^2 (7.6)^{(2/2.6)}}{4\,(0.25)\,(4)^{(4.6/2.6)}}\; 10^8$$

$$= 0.00937$$

From Appendix 2

A2.2 EE core data



Core type (A) (mm)	Geometrical constant K_g cm^5	Geometrical constant K_{gfe} cm^x	Cross-sectional area A_c (cm^2)	Bobbin winding area W_A (cm^2)	Mean length per turn MLT (cm)	Magnetic path length l_m (cm)	Core weight (g)
EE22	$8.26 \cdot 10^{-3}$	$1.8 \cdot 10^{-3}$	0.41	0.196	3.99	3.96	8.81
EE30	$85.7 \cdot 10^{-3}$	$6.7 \cdot 10^{-3}$	1.09	0.476	6.60	5.77	32.4
→ EE40	0.209	$11.8 \cdot 10^{-3}$	1.27	1.10	8.50	7.70	50.3
EE50	0.909	$28.4 \cdot 10^{-3}$	2.26	1.78	10.0	9.58	116

Evaluate ac flux density B_{max}

Eq. (14.41):

$$B_{max} = \left[10^8 \, \frac{\rho \lambda_1^2 I_{tot}^2}{2K_u} \, \frac{(MLT)}{W_A A_c^3 l_m} \, \frac{1}{\beta K_{fe}} \right]^{\left(\frac{1}{\beta + 2}\right)}$$

Plug in values:

$$B_{max} = \left[10^8 \, \frac{(1.724 \cdot 10^{-6})(800 \cdot 10^{-6})^2 (14.4)^2}{2\,(0.25)} \, \frac{(8.5)}{(1.1)(1.27)^3(7.7)} \, \frac{1}{(2.6)(7.6)} \right]^{(1/4.6)}$$

$$= 0.23 \text{ Tesla}$$

This is less than the saturation flux density of approximately 0.35 T

Evaluate turns

Choose n_1 according to Eq. (14.42):

$$n_1 = \frac{\lambda_1}{2B_{max}A_c}\ 10^4$$

$$n_1 = 10^4 \frac{(800 \cdot 10^{-6})}{2(0.23)(1.27)}$$

$$= 13.7 \text{ turns}$$

Choose secondary turns according to desired turns ratios:

$$n_2 = \frac{5}{110} n_1 = 0.62 \text{ turns}$$

$$n_3 = \frac{15}{110} n_1 = 1.87 \text{ turns}$$

Rounding the number of turns

To obtain desired turns ratio of

110:5:15

we might round the actual turns to

22:1:3

Increased n_1 would lead to

- Less core loss
- More copper loss
- Increased total loss

Loss calculation
with rounded turns

With $n_1 = 22$, the flux density will be reduced to

$$B_{max} = \frac{(800 \cdot 10^{-6})}{2\,(22)\,(1.27)}\,10^4 = 0.143 \text{ Tesla}$$

The resulting losses will be

$$P_{fe} = (7.6)(0.143)^{2.6}(1.27)(7.7) = 0.47 \text{ W}$$

$$P_{cu} = \frac{(1.724 \cdot 10^{-6})(800 \cdot 10^{-6})^2(14.4)^2}{4\,(0.25)}\,\frac{(8.5)}{(1.1)(1.27)^2}\,\frac{1}{(0.143)^2}\,10^8$$

$$= 5.4 \text{ W}$$

$$P_{tot} = P_{fe} + P_{cu} = 5.9 \text{ W}$$

Which exceeds design goal of 4 W by 50%. So use next larger core size: EE50.

Calculations with EE50

Repeat previous calculations for EE50 core size. Results:

$B_{max} = 0.14$ T, $n_1 = 12$, $P_{tot} = 2.3$ W

Again round n_1 to 22. Then

$B_{max} = 0.08$ T, $P_{cu} = 3.89$ W, $P_{fe} = 0.23$ W, $P_{tot} = 4.12$ W

Which is close enough to 4 W.

Wire sizes for EE50 design

Window allocations

$$\alpha_1 = \frac{I_1}{I_{tot}} = \frac{5.7}{14.4} = 0.396$$

$$\alpha_2 = \frac{n_2 I_2}{n_1 I_{tot}} = \frac{5}{110}\,\frac{66.1}{14.4} = 0.209$$

$$\alpha_3 = \frac{n_3 I_3}{n_1 I_{tot}} = \frac{15}{110}\,\frac{9.9}{14.4} = 0.094$$

Wire gauges

$$A_{w1} = \frac{\alpha_1 K_u W_A}{n_1} = \frac{(0.396)(0.25)(1.78)}{(22)} = 8.0 \cdot 10^{-3}\ \mathrm{cm}^2$$

$$\Rightarrow \text{AWG \#19}$$

$$A_{w2} = \frac{\alpha_2 K_u W_A}{n_2} = \frac{(0.209)(0.25)(1.78)}{(1)} = 93.0 \cdot 10^{-3}\ \mathrm{cm}^2$$

$$\Rightarrow \text{AWG \#8}$$

$$A_{w3} = \frac{\alpha_3 K_u W_A}{n_3} = \frac{(0.094)(0.25)(1.78)}{(3)} = 13.9 \cdot 10^{-3}\ \mathrm{cm}^2$$

$$\Rightarrow \text{AWG \#16}$$

Might actually use foil or Litz wire for secondary windings